# Pre-Inflationary Spacetime in String Cosmology

Brett McInnes

National University of Singapore
email: matmcinn@nus.edu.sg

ABSTRACT

Seiberg and Witten have shown that the non-perturbative stability of string physics on conformally compactified spacetimes is related to the behaviour of the areas and volumes of certain branes as the brane is moved towards infinity. If, as is particularly natural in quantum cosmology, the spatial sections of an accelerating cosmological model are *flat* and compact, then the spacetime is on the brink of disaster: it turns out that the version of inflationary spacetime geometry with toral spatial sections is marginally stable in the Seiberg-Witten sense. The question is whether the system remains stable *before* and *after* Inflation, when the spacetime geometry is distorted away from the inflationary form but still has flat spatial sections. We show that it is indeed possible to avoid disaster, but that requiring stability at all times imposes non-trivial conditions on the spacetime geometry of the early Universe in string cosmology. This in turn allows us to suggest a candidate for the structure which, in the earliest Universe, forbids cosmological singularities.

# 1. Toral String Cosmology

There are many reasons to suspect that, in string theory, the most natural topology for the spatial sections of the Universe is that of a *flat, compact* three-manifold — that is, the topology of a torus or a certain finite quotient of a torus[1]. For example, consider the recent work of Ooguri, Vafa, and Verlinde [[4]; see also [5]], who have related the topological string partition function to the Hartle-Hawking "wave function of the Universe" [6]. Ooguri et al begin by emphasising that it is natural, in the context of quantum gravity, to assume that the spatial sections of our universe are *compact*, with their initial size parameters being regarded as moduli to be selected by a wave function with amplitudes peaked at the appropriate values. Toral cosmology [7][8] is particularly natural in this picture, because the size of the torus is decoupled from the spacetime length scale — it is not at all constrained by classical general relativity, and so it can *only* be fixed by quantum gravity.

The idea of fixing the initial size of the Universe by means of the wave function of the Universe has in fact been explored concretely in [9][10][11] [in the spherical case] and [12] [in the toral case]; see also [13] for related ideas. In particular, when the Hartle-Hawking wave function [as modified by Firouzjahi, Sarangi, and Tye [9][10][11]] is applied to the problem of predicting the initial size of a Universe created from "nothing", we find in the *toral* case that the predicted scale for the initial size of the Universe is about the string scale [12]. This is a self-consistency check for our claim that string theory favours toral sections: one does *not* reach this conclusion if locally spherical sections are assumed.

Toral cosmology also arises in the celebrated work of Brandenberger and Vafa on *string gas cosmology* [see [14], and [15] for a recent extensive review]. There the crucial arguments based on T-duality and string winding modes make explicit use of the toral topology of three-dimensional space. Still another string-cosmology investigation in which toral sections play a basic role is the recent work of McGreevy and Silverstein [16].

Finally, of course, such spacetimes are also compatible with the cosmological observations pointing to spatial flatness [17][18], though, if there was a period of Inflation, one does not expect to be able to observe the non-trivial topology *directly*. [It seems that this cannot be done at the *present* time [19], and inflationary theory suggests that it will probably never be possible; see however [20].] However, the toral structure may well have been "observable" in the very early Universe, and, as we shall see, this can yield crucial clues as to the spacetime geometry at that time.

Our Universe appears to have a remarkable property: it passes through phases in which its local geometry closely resembles that of de Sitter spacetime. This appears to have happened in the past, during an *inflationary* era, and it seems likely that it will happen again, due to the current presence of dark energy.

In toral cosmology, the inflationary era is described by *Spatially Toral de Sitter* [STdS] spacetime. If the torus has side length $2\pi K$ at time $t = 0$, and if the spacetime curvature is $-1/L^2$ [in signature $(+ - - -)$], corresponding to a positive vacuum energy density $3/8\pi L^2$, then the metric is

$$g(\text{STdS})(K, L)_{+---} = \text{dt}^2 - K^2 e^{(2t/L)} [\text{d}\theta_1^2 + \text{d}\theta_2^2 + \text{d}\theta_3^2], \qquad (1)$$

---

[1]If we use a *quotient* of a torus, the quotient should be non-singular [1] for the "large" dimensions, though not necessarily for the "small" [2][3]. Henceforth we refer to *all* such topologies as being "toral".

where the torus is parametrized by angles; here t runs from $-\infty$ to $+\infty$. This spacetime is *locally* identical to the familiar global de Sitter spacetime, but its global structure is very different: the spacetime topology[2] is $\mathbb{R} \times \mathrm{T}^3$, whereas of course global simply connected de Sitter spacetime has topology $\mathbb{R} \times \mathrm{S}^3$, where $\mathrm{S}^3$ is the three-sphere. In this work we shall explore the *physical* differences that follow from this *topological* difference.

Physically, STdS spacetime differs from global [simply connected] de Sitter spacetime in two fundamental ways: one pertaining to the past, the other to the future.

First, as is well known, global de Sitter spacetime is geodesically complete. *This is not true of STdS spacetime* [21][22], which is incomplete in the past. Furthermore, the incomplete region immediately develops into a curvature singularity if any conventional matter is introduced; and, worse yet, this behaviour is generic, in a strong sense that we shall explain. But one of the main objectives of string cosmology is to *solve* the singularity problem in cosmology. Clearly this contradiction needs to be addressed. It implies that there must have been a *pre-inflationary era* with a spacetime geometry which must have involved significant deformations of the STdS metric. In fact, the Andersson-Galloway theorem, mentioned briefly in [12] and to be discussed in detail here, tells us that the deformation must mimic the effects of "matter" which *apparently* has negative energy density.

The second difference is more obvious: the future conformal boundary of global simply connected de Sitter spacetime is a copy of $\mathrm{S}^3$, while that of STdS is a copy of $\mathrm{T}^3$. Now in string theory it is known that the structure of conformal infinity has profound consequences for the perturbative and especially the non-perturbative stability of the theory. In particular, Seiberg and Witten [23] showed explicitly that having a spherical structure at infinity plays a role in preventing instability due to the nucleation of "large branes" near infinity. If one replaces the sphere at infinity by some other space, then the theory is in danger of becoming inconsistent. This Seiberg-Witten mechanism has been applied to topologically non-trivial black hole spacetimes in [24]. More importantly here, it has recently been applied to *cosmology* by Maldacena and Maoz [25], and their work has been extended in various ways in [26][27][1][28].

The system remains stable near infinity as long as the conformal structure at infinity continues to be represented by a metric of positive *scalar curvature*. If the scalar curvature at infinity becomes negative, then the system is definitely unstable. Thus the borderline between stable and unstable cases *runs somewhere through the space of conformally compactified manifolds with zero scalar curvature at infinity*. It follows that replacing global de Sitter spacetime with its toral version immediately pushes the spacetime much closer to the brink of instability.

We have seen already that the spacetime geometry of the pre-inflationary era must be a strongly deformed version of the STdS metric, so that string cosmology can be non-singular. Similarly, of course, the *post*-inflationary era must have a geometry which differs substantially from that of STdS, since radiation and matter can no longer be ignored in that era. Given that STdS is already perilously close to being unstable in the Seiberg-Witten sense, *it is clearly imperative to verify that the pre- and post-inflationary distortions of the STdS metric do not give rise to a drastic non-perturbative instability in*

---

[2]Note that K can be scaled to unity by a simple translation of the time coordinate. This is however no longer true if matter is inserted; see below for the physical significance of K.

*string theory.* This is one objective of the present work.

In order to explore this question, we need to specify more precisely the ways in which the STdS geometry is deformed in the pre- and post-inflationary eras. This is most straightforward in the post-inflationary case, so we shall discuss that case first. We find that the relevant brane action *increases* towards infinity after Inflation; hence it will always be positive, indicating stability, provided that it was positive when Inflation *ended*. But conditions at the end of Inflation are determined by conditions at its beginning.

The question of Seiberg-Witten stability at the beginning of Inflation is much more difficult, because of uncertainties as to the geometry at that time. Our strategy is to rely on the constraints imposed by the fact that string cosmology should be non-singular, and also by the very fact that *it is not easy to get Inflation to start at all* in string cosmology.

This point has recently been stressed by Linde, who argues [29][30] that when Inflation is embedded in string cosmology [31], it begins at a fairly large length scale. But in string cosmology, the Universe can be as small as the string length scale, which may be two orders of magnitude smaller. Linde points out that discrepancies like this could be a problem, because even if the Universe is born in some very uniform state, it cannot in general be expected to remain homogeneous during the time when it expands from the string[3] to the inflationary length scale. Linde proposes to solve this problem precisely by assuming that the spatial sections of the Universe are toral. Under certain circumstances, this allows all parts of the Universe to remain in causal contact during the pre-Inflationary phase; then homogeneity can be maintained by means of chaotic mixing [32] until Inflation is ready to begin. Using this idea we can construct a simple explicit model of the pre-inflationary era.

It turns out that the Seiberg-Witten brane action actually tends to *decrease* with the expansion during the pre-inflationary era. Thus, even though its initial value is positive, there is a danger that it will indeed become negative before the end of Inflation. Because the inflationary era is long, measured in units of the inflationary scale, even a gentle rate of decline can lead to instability. This argument, combined with others, allows us to put strong constraints on the spacetime geometry of the pre-inflationary era, and on the nature of the [effective] field with negative energy density. For example, we can rule out Casimir energy as a means of resolving the initial singularity. The best candidate appears to be the constraint field which appears in the Gabadadze-Shang "classically constrained gravity" theory [33][34]. This field is non-dynamical by its very nature, and this naturally resolves the usual objections to negative "energies".

An alternative way of constraining the pre-inflationary geometry was suggested in [12]. That approach is however based on a specific modification of the Hartle-Hawking wavefunction, the FST wave function [9] [see also [35][36]]. Here we take a much more conservative approach based strictly on stability. It is pleasing that, in fact, the results of the two approaches are consistent.

The principal new results of the present work are the following. First, we show explicitly [Section 2] that there is no danger of Seiberg-Witten instability in toral string cosmology as long as conventional matter dominates. Next, while it is well known that FRW models with *exactly* flat spatial sections must apparently violate energy conditions in order to be non-singular, in Section 3 we use the results of Andersson and Galloway [21]

---

[3]Actually, Linde prefers a scenario in which the Universe is born at the Planck scale.

4

to argue that this conclusion is particularly strong in the toral case: it remains valid even if we allow the large distortions of spacetime geometry to be expected in the very early Universe. Guided by this, in Section 4 we construct an explicit one-parameter family of non-singular spacetimes which allow us to investigate Seiberg-Witten instability in the pre-inflationary era. Section 5 uses this explicit geometry to identify the specific structure which allows the pre-inflationary spacetime to be non-singular: the Gabadadze-Shang constraint field.

## 2. Stability After Inflation

In this section we shall show that, even with toral spatial sections, there is no danger of non-perturbative string instability in the post-inflationary era, provided that there is no such instability during and before Inflation. We begin by briefly describing the toral versions of the relevant classical cosmological spacetimes.

The post-inflationary Universe contains matter described by a small positive cosmological constant [of, at least approximately, the current value] together with various kinds of radiation and matter. To discuss questions of stability it is simplest to follow cosmological practice and regard this additional matter as a general "fluid" with an equation of state of the standard form

$$p = w\rho, \tag{2}$$

where p represents the pressure, $\rho$ the density, and where w is a constant. It will be convenient to represent this constant in terms of another constant $\epsilon$, defined by

$$\epsilon = 3(1 + w). \tag{3}$$

We emphasise that this "fluid" is to be *superimposed* on a positive vacuum energy density of magnitude $3/8\pi L^2$: we intend to introduce this substance into Spatially Toral de Sitter spacetime.

Thus for example if we insert non-relativistic matter [zero pressure] into the STdS spacetime and allow it to act [via the Einstein equations], we shall have $\epsilon = 3$, while $\epsilon = 4$ corresponds to ordinary radiation; values of $\epsilon$ from 1 to 1.5 arise if we are interested in the back-reaction induced by domain walls [37] on the STdS geometry, and so on. We stress however that we are *not* primarily interested in using this "fluid" to violate the Strong Energy Condition; in the cases of principal interest to us, the acceleration is due to the negative contribution made by the STdS cosmological constant to the total pressure. The "fluid" satisfies the Strong Energy Condition for all values of $\epsilon$ greater than or equal to 2.

We shall consider Friedmann cosmological models with metrics of the form

$$g = dt^2 - K^2 a(t)^2[d\theta_1^2 + d\theta_2^2 + d\theta_3^2]; \tag{4}$$

this generalizes the STdS metric in an obvious way. Adding the energy density of the "fluid" to that of the initial STdS space, one can solve the Einstein equations to obtain

$$g_s(\epsilon, K, L)_{+---} = dt^2 - K^2 \sinh^{(4/\epsilon)}\left(\frac{\epsilon t}{2L}\right)[d\theta_1^2 + d\theta_2^2 + d\theta_3^2]; \tag{5}$$

here the s refers to the sinh function. The density of the "fluid" turns out to be given by

$$\rho = \frac{3}{8\pi L^2 a^\epsilon}. \tag{6}$$

5

If $\epsilon = 3$, we should have the local metric for a spacetime containing non-relativistic matter and a de Sitter vacuum energy, and indeed $g_s(\epsilon, K, L)_{+---}$ reduces in this case — purely locally, of course — to the classical Heckmann metric [see [38] for a recent discussion]. In the general case it agrees [again locally] with the results reported in [39]. For large t we have

$$g_s(\epsilon, 2^{2/\epsilon} K, L)_{+---} \approx dt^2 - K^2 e^{2t/L}[d\theta_1^2 + d\theta_2^2 + d\theta_3^2], \tag{7}$$

which is the STdS metric given in equation (1); notice that $\epsilon$ effectively drops out. Thus our metric is "asymptotically STdS", for all $\epsilon$.

The matter content of these spacetimes does not behave as simply as one might expect. For while it is true that both components, the vacuum energy and the "fluid", separately have constant equation-of-state parameters, *their combination does not*: if we denote by W the ratio of the total pressure to the total density, then we find

$$W = -1 + \frac{\epsilon}{3} \operatorname{sech}^2\left(\frac{\epsilon t}{2L}\right). \tag{8}$$

Thus W decreases, from a limiting value of $-1 + (\epsilon/3)$ as t tends to zero, to its asymptotic STdS value $-1$. The Strong Energy Condition is violated if $W < -1/3$, so [leaving aside $t = 0$ itself, which we shall discuss shortly] the SEC holds in the early universe provided that $-1 + (\epsilon/3) > -1/3$, which just means that $\epsilon$ should be greater than 2. In this case there is a transition from deceleration to acceleration, as is observed in our Universe [40]; so this is the case of real physical interest.

This transition time occurs when t is precisely such that $W = -1/3$, that is,

$$t_{\mathrm{TRANS}} = \frac{2L}{\epsilon} \cosh^{-1}\left(\sqrt{\epsilon/2}\right). \tag{9}$$

From equation (5) one can compute the circumference C(t) of the spatial torus [along any component circle]; at the transition time,

$$C(t_{\mathrm{TRANS}}) = 2\pi K \sinh^{(2/\epsilon)}\left(\frac{\epsilon t_{\mathrm{TRANS}}}{2L}\right) = 2\pi K\left[(\epsilon/2) - 1\right]^{1/\epsilon}. \tag{10}$$

This gives a physical interpretation of the length K: for example, in the case of radiation [$\epsilon = 4$], K is precisely the radius of the spatial torus at the time of transition from deceleration to acceleration. The point we wish to stress is that K *does* have a concrete physical meaning in these spacetimes: it cannot simply be "scaled away" by changing the time coordinate, as can be done in pure STdS spacetime.

These spacetimes have a genuine [curvature] singularity at $t = 0$; for example, the scalar curvature is given by

$$R(g_s(\epsilon, K, L)_{+---}) = -\frac{12}{L^2} + \frac{3}{L^2}(\epsilon - 4)\operatorname{cosech}^2\left(\frac{\epsilon t}{2L}\right); \tag{11}$$

this tends to $-12/L^2$ as t tends to infinity, the correct asymptotic de Sitter value in this signature, but it clearly diverges as t tends to zero [except in the $\epsilon = 4$ case, which corresponds to radiation and hence to a traceless stress-energy tensor which does not contribute to the scalar curvature; but this case is still singular, as one sees by examining

6

other curvature invariants]. We stress that the spacetime is singular for *all* $\epsilon$, including values such that the Strong Energy Condition is *violated at all times.* We shall discuss the significance of this basic fact in detail in a later section.

Following Seiberg and Witten [23], we now turn to the question of the *non-perturbative* stability of these spacetimes. The basic point here is that branes, being extended objects, can be extremely sensitive to the geometry of the spaces in which they propagate. It follows that modifying that geometry can have major and unexpected consequences.

Seiberg and Witten consider the Euclidean version of the AdS/CFT setup; this means that we are dealing with a space with a well-defined conformal boundary, so that the geometry asymptotically resembles that of hyperbolic space. Let us introduce a 4-form field on a Euclidean, asymptotically hyperbolic four-manifold, and consider the nucleation of Euclidean BPS 2-branes. The brane action consists of two terms: a positive one contributed by the brane tension, but also a *negative* one induced by the coupling to the antisymmetric tensor field. As the first term is proportional to the area of the brane, while the second is proportional to the volume enclosed by it, we have in four dimensions

$$ S \ = \ T(A \ - \ \frac{3}{L} V), \tag{12} $$

where T is the tension, A is the area, V the volume enclosed, and L is the background asymptotic curvature radius.

The stability of this system is determined by a *purely geometric* question: can the area of a brane always grow quickly enough to keep the action positive? If this is not the case, then we have a serious instability, which has been described by Maldacena and Maoz [25] as a *pair-production instability* for branes. In ordinary [non-compactified] hyperbolic space the action is strictly positive, *but in certain distorted versions of hyperbolic space it is not.* Seiberg and Witten found that this form of instability is never a problem near infinity provided that hyperbolic space is deformed only mildly: to be precise, the action remains positive near infinity as long as the conformal structure at infinity continues to be represented by a metric of positive scalar curvature [as is the case for hyperbolic space itself, which has a *spherical* conformal boundary]. If the deformation is such that the scalar curvature at infinity becomes negative, then the system is definitely unstable. The borderline case, where the scalar curvature at infinity is zero, *is precisely the one of interest to us here*, since the torus is flat, therefore scalar-flat. In this case, the question of non-perturbative stability can only be settled by a detailed examination; explicit examples of stable spaces with zero scalar curvature at infinity have been given by Kleban et al [27], but explicit examples, with flat boundaries, which are definitely *unstable* have also been given in [28]. The danger of instability is therefore very real in this case.

Let us begin by examining the relevant Euclidean version of STdS spacetime in detail. Because of the central role of the conformal boundary in this discussion, it is natural to study the Euclidean versions of our spacetimes by complexifying *conformal* time [as in [28] and the recent work of Banks et al [41]]. The Euclidean version of (1) is then

$$ g(\text{STdS})(\text{K, L})_{++++} \ = \ \frac{L^2}{\eta_+^2} [\,d\eta_+^2 \ + \ d\theta_1^2 \ + \ d\theta_2^2 \ + \ d\theta_3^2]. \tag{13} $$

Here $\eta_+$, the Euclidean [dimensionless] conformal time, takes its values in the open interval $(-\infty, 0)$. [K is now hidden in the definition of $\eta_+$, which is given by $d\eta_+ \ = \ dt/Ke^{t/L}$.]

Conformal infinity is defined by extending to $\eta_+ = 0$, and clearly it is a copy of $T^3$ with its standard conformal structure based on a *flat* metric. Thus spatially toral de Sitter spacetime lies close to the region of Seiberg-Witten instability. The question is: how close?

One might think that, since the metric (13) differs from that of hyperbolic space only globally, there should be no danger of instability here. But branes, being extended objects, are sensitive to certain global geometric features: the action involves non-local quantities, namely area and volume, the growth of which can be strongly affected by the global structure of the ambient manifold. *Changing the global structure might well drive the system closer to the unstable case.* This does in fact happen in the STdS case, though fortunately not to the extent that any instability is actually induced.

We can compute the brane action [equation (12)] for the metric in equation (13) directly: the area form is just $-\,L^3\,d\theta_1\,d\theta_2\,d\theta_3/\eta_+^3$ [since $\eta_+$ is negative] and the volume form is $L^4\,d\eta_+\,d\theta_1\,d\theta_2\,d\theta_3/\eta_+^4$. If we imagine creating a brane at $\eta_+ = E_+ < 0$ and then moving it towards the boundary, the action as a function of $\eta_+$ is

$$S_{STdS}(\eta_+) \;=\; T\left\{ -\,\frac{8\,\pi^3\,L^3}{\eta_+^3} \;-\; \frac{3}{L}\int_{E_+}^{\eta_+} \frac{8\,\pi^3\,L^4\,d\eta_+}{\eta_+^4} \right\} \;=\; -\,\frac{8\pi^3\,L^3\,T}{E_+^3} \;>\; 0. \qquad (14)$$

Thus the action is *a positive constant.* Notice that this calculation only makes sense because we have compactified the spatial sections; brane physics is indeed sensitive to the distinction between ordinary de Sitter spacetime and its spatially toral version.

Clearly the STdS spacetime is not unstable in the Seiberg-Witten sense; however, it might not be difficult to render it unstable by means of some arbitrarily small deformation, since even a small negative contribution to the slope of the action could eventually lead, as we move towards the boundary, to negative values for the action itself. If it does become negative, then we have a non-perturbative instability of the system. Explicit examples of this have been given in [25] [for negatively curved boundaries] and [28] [for flat boundaries].

Let us see how this works for the spacetimes we obtained earlier by introducing various kinds of matter into STdS spacetime. As in the case of STdS itself, we use conformal time. The Euclidean version of our metric $g_s(\epsilon, K, L)_{+---}$ [equation (5)] that is asymptotic to $g(STdS)(K, L)_{++++}$ [equation (13)] is simply[4]

$$g_s(\epsilon, K, L)_{++++} \;=\; K^2\,\sinh^{(4/\epsilon)}\!\left(\frac{\epsilon\,u(\eta_+)}{2\,L}\right)\left[\,d\eta_+^2 \;+\; d\theta_1^2 \;+\; d\theta_2^2 \;+\; d\theta_3^2\,\right]. \qquad (15)$$

Here the Euclidean analogue of conformal time, $\eta_+$, is defined on the interval $(-\,H(L/K, \epsilon), 0)$, where, as in equation (13), conformal infinity is at $\eta_+ = 0$, and where

$$H(L/K, \epsilon) \;=\; \frac{2\,L}{K\epsilon}\int_0^\infty \frac{dx}{\sinh^{(2/\epsilon)}(x)}. \qquad (16)$$

This quantity is finite if and only if $\epsilon > 2$; as we saw, this is the observationally interesting range in which the Universe undergoes a period of deceleration before beginning

---

[4]Strictly speaking, K should be re-scaled by a factor $2^{(2/\epsilon)}$ for this statement to be precisely correct.

to accelerate. Finally, $u(\eta_+)$ is the function of $\eta_+$ which tends to zero as $\eta_+$ tends to $-H(L/K, \epsilon)$ and solves

$$\frac{du}{d\eta_+} = K \sinh^{(2/\epsilon)}\left(\frac{\epsilon \, u(\eta_+)}{2\,L}\right). \tag{17}$$

The metric $g_s(\epsilon, K, L)_{++++}$ is an example of an asymptotically hyperbolic Riemannian metric; we can think of it as a deformation of the Euclidean STdS metric; as with the latter, the underlying manifold is the product of an interval with $T^3$. If we create a brane at $\eta_+ = E_+$ in the interval $(-H(L/K, \epsilon), 0)$, and consider the effects of moving it towards infinity [that is, towards $\eta_+ = 0$], the relevant action is

$$S(g_s(\epsilon, K, L)_{++++} \, ; \, \eta_+) = 8\pi^3 \, TK^3 \left[ \sinh^{(6/\epsilon)}\left(\frac{\epsilon \, u(\eta_+)}{2L}\right) - \frac{3}{L} \int_{u(E_+)}^{u} \sinh^{(6/\epsilon)}\left(\frac{\epsilon \, u}{2L}\right) du \right]. \tag{18}$$

In the case of non-relativistic matter $[\epsilon = 3]$ this can be evaluated, the result being

$$S(g_s(3, K, L)_{++++} \, ; \, \eta_+) = 4\pi^3 \, TK^3 \left[ \frac{3u(\eta_+)}{L} + e^{-3u(\eta_+)/L} - 1 \right.$$
$$\left. + \sinh\left(\frac{3u(E_+)}{L}\right) - \frac{3u(E_+)}{L} \right]. \tag{19}$$

A sketch of the graph of the right side as a function of $u(\eta_+)$ shows that this is *never negative*, and so there is no danger of Seiberg-Witten instability here.

In fact this is true for all values of $\epsilon$. A straightforward calculation shows that the derivative of the brane action in the general case is given by

$$\frac{d}{d\eta_+} S(g_s(\epsilon, K, L)_{++++} \, ; \, \eta_+) = \frac{24\pi^3 \, TK^4}{L} \sinh^{(8/\epsilon)}\left(\frac{\epsilon \, u(\eta_+)}{2L}\right) \left[ \coth\left(\frac{\epsilon \, u(\eta_+)}{2L}\right) - 1 \right]. \tag{20}$$

This is obviously positive. Since

$$S(g_s(\epsilon, K, L)_{++++} \, ; \, E_+) = 8\pi^3 \, TK^3 \sinh^{(6/\epsilon)}\left(\frac{\epsilon \, u(E_+)}{2L}\right) \tag{21}$$

is also positive, it is clear that, *provided we are in the regime where this metric is valid*, the action will never be negative — there is no Seiberg-Witten instability here for any value of $\epsilon$. However, we have to be cautious, for we know that, in fact, the metric we are studying here is not valid in the Inflationary era. Therefore, a more precise statement is as follows: *the action will never be negative if it is positive when Inflation ends*, since it cannot decrease after that point.

Thus string cosmology *is* stable, *provided* that it is stable up to the end of Inflation. The future of spacetime is secure. The question is whether the same can be said of its *past*.

## 3. The Past of Spatially Toral de Sitter and its Relatives

In this section, we discuss the global theory of accelerating spacetimes with toral spatial sections. It is well known that *exact* FRW cosmologies with *exactly* flat spatial sections must violate energy conditions if they are to be non-singular. Less well known is the fact that these violations are inevitable under *very much more general* assumptions.

The problem of avoiding singularities in general accelerating cosmologies is of course related to the question as to whether Inflation must necessarily have a beginning. This has been discussed extensively in the literature: see especially [42]. These discussions are based on FRW cosmological models with spatial sections that are *exactly flat* and remain flat arbitrarily far back in time. In reality, of course, the spacetime geometry could be severely distorted in a pre-inflationary era, so the question of singularity avoidance remained open. Recently, however, the work of Andersson and Galloway [21] has given us a new insight into these results, precisely in the case of interest to us here — that of compact spatial sections.

Consider again Spatially Toral de Sitter spacetime, with its metric given in equation (1). Contrary to appearances, this spacetime is actually geodesically incomplete: past-directed timelike geodesics can wind around the torus in such a way that they reach $t = -\infty$ in a *finite* amount of proper time. In this, of course, STdS differs radically from the version of de Sitter spacetime with locally spherical spatial sections, which is complete. The incompleteness here is relatively harmless, in that it does not involve divergent curvatures; but we saw in the previous section that the introduction of a wide variety of kinds of matter into STdS spacetime *inevitably* causes the incompleteness to develop into a true curvature singularity. In this, too, STdS differs from locally spherical de Sitter spacetime, which remains non-singular even if [a small amount of] conventional matter is introduced. Take, for example, the spacetime corresponding to the Euclidean "barrel" discussed in [10], which is "spatially spherical de Sitter plus a small amount of radiation": it is non-singular.

We can summarize this state of affairs by saying that the toral versions of de Sitter are "more singularity-prone" than the locally spherical versions.

To understand the reasons for this crucial property, which is of basic importance for string cosmology, we must use the Andersson-Galloway theorem [21][22]. The theorem may be stated as follows; we refer the reader to the Appendix for details of the terminology and a brief commentary on this remarkable result.

THEOREM [Andersson-Galloway]: *Let* $M_{n+1}$, $n \leq 7$, *be a globally hyperbolic (n+1)-dimensional spacetime with a regular future spacelike conformal boundary* $\Gamma^+$. *Suppose that the Null Ricci Condition is satisfied and that* $\Gamma^+$ *is compact and orientable. If* $M_{n+1}$ *is past null geodesically complete, then the first homology group of* $\Gamma^+$, $H_1(\Gamma^+, \mathbb{Z})$, *is pure torsion.*

Here the *Null Ricci Condition* is the requirement that, for all null vectors $k^\mu$, the Ricci tensor should satisfy

$$R_{\mu\nu} k^\mu k^\nu \geq 0. \tag{22}$$

The STdS spacetime has a regular future conformal boundary; this boundary is compact and orientable, since it is just the torus $T^3$. The Null Ricci Condition is satisfied, since the spacetime is an Einstein space. But the first homology group of $T^3$ is certainly *not* pure torsion [that is, not every element is of finite order]: it is isomorphic to $\mathbb{Z} \oplus \mathbb{Z} \oplus \mathbb{Z}$. Thus the spacetime *had* to be null geodesically incomplete. [In fact, as we mentioned above, it is also timelike geodesically incomplete; see [22].]

The surprising and beautiful feature of the Andersson-Galloway theorem is that this same conclusion holds *no matter how we distort the geometry at early times*, as long as

the Null Ricci Condition continues to hold. That is, if we introduce *any* kind of matter at early times such that the NRC continues to hold — if the Einstein equations are valid, then this just means that the Null Energy Condition [NEC] is satisfied — then the resulting cosmology will necessarily be null incomplete to the past, no matter how distorted the spatial geometry may have been at that time. In other words, the Andersson-Galloway theorem is a *singularity theorem*: as with the classical singularity theorems, the conclusion does not depend on assumptions about local isotropy or homogeneity or indeed on whether we assume a Friedmann geometry at all. The prediction of a singularity is very robust.

The Andersson-Galloway theorem is fundamental to string cosmology, because one of the latter's major objectives is precisely to solve the problem of the initial singularity in conventional cosmology. The fact that the theorem makes such robust predictions means that it tells us something very fundamental about these cosmologies.

What the Andersson-Galloway theorem is telling us — see the Appendix for a detailed discussion of this — is that *the Null Ricci Condition [NRC] must be violated in the early history of a non-singular accelerating string cosmology with toral sections*.

Once we accept that NRC violation is inevitable here, we can use this fact to construct an explicit spacetime metric for the pre-inflationary era, and then use it to check for Seiberg-Witten instability in that era.

## 4. Stability Before Inflation: Not Guaranteed

We have stressed that the *only* way to solve the singularity problem of accelerating string cosmologies is to violate the Null Ricci Condition [NRC], or the Null Energy Condition [NEC] if we assume the validity of the Einstein equations. We shall now use this fact as a guide to the geometry of the earliest history of a string cosmology: the pre-inflationary era.

A fundamental matter field which genuinely violates the NEC can be hard to handle, for a variety of reasons ranging from simple field-theoretic instability to conflict with the holographic principle: see for example [43][44][28]. Here we shall take the conservative view that true NEC violation is not acceptable in cosmology. Purported observations of violations of the NEC at late times [45] may have other explanations; see for example [46][47][48].

Instead of using fundamental matter fields violating the NEC, we shall consider two alternative approaches. The first possibility is that violations of the NRC in the early Universe are due to energy densities which are *negative*, not because of the presence of some exotic matter field, but rather because unusual topology or geometry or non-dynamical structures had equally unusual effects at that time. A well-known example of such phenomena is that the topology we are assuming here could lead to Casimir effects; see for example [49]. Again, it is widely agreed that negative-tension branes are acceptable if they are not dynamical. More interestingly for our purposes here, Gabadadze and Shang have pointed out that, in their "classically constrained" gravity theory [33], the constraint field can [in certain cases [34]] give rise to a negative energy density. This is again physically acceptable since, by its very definition, a constraint field will not be dynamical. We shall return to this example below.

A second possibility [which can be combined with the first] is to exploit the distinction

between the NRC and the NEC. The former is of course a purely geometric condition [see (22) above], while the latter refers only to items such as pressure and energy densities. The two are linked by the gravitational field equation. They are equivalent if the Einstein equations hold exactly, but of course this is a highly questionable assumption in the *early* Universe. In braneworld models [50] there are explicit corrections to the Einstein equation which allow the NRC to be violated *while every matter field satisfies the NEC*, so that one says that the NEC is "effectively" violated [51][28]. The distinction between the NEC and the NRC is also important when one studies the relationship between brane and bulk when the brane cosmology accelerates [52][53]. Similarly, the NEC and the NRC can be usefully different in certain Gauss-Bonnet theories [54][55] and also in scalar-tensor theories [56][57]; note that scalar-tensor theories of precisely this kind do arise in certain string cosmologies [58]. Notice that one can think about the Gabadadze-Shang theory [33][34] in this way also, since this theory certainly modifies the Einstein equation.

In short, violation of the Null *Energy* Condition is not necessarily the issue here. The real question is this: does violating the Null *Ricci* Condition have any direct unwelcome consequences, even if the NEC is *not* violated?

This is where Seiberg-Witten instability is central. For as we have stressed, this particular form of instability depends only on geometric data associated with branes [rates of growth of volume and area]. Thus, unlike more familiar sources of instability, Seiberg-Witten instability could be directly related to purely geometric conditions like the NRC, and be entirely independent of the gravitational field equations, whatever form the latter may take for strong fields. That is, we might have to confront Seiberg-Witten instability *even under conditions where the null energy condition is fully respected*. This in fact proves to be the case.

In order to investigate the role of Seiberg-Witten instability in the early Universe, we need an explicit metric describing the geometry of the pre-inflationary era. We shall now construct a family of [necessarily NRC-violating] relevant metrics, guided by the special physics of string cosmology.

We shall assume that during the pre-inflationary era, as in the inflationary era itself, there is a dominant form of matter with positive energy density and which *satisfies* the NEC; this could be the inflaton itself. We assume that the energy density of this component varies sufficiently slowly that it can be approximated by the inflationary energy density $3/8\pi L_{inf}^2$, where the inflationary length scale $L_{inf}$ is of course very much smaller than the scale L we used earlier to describe the late-time acceleration of the Universe.

We shall assume that the NRC is approximately equivalent to the NEC during the inflationary era, but that they differ significantly in the pre-inflationary era. Thus, assuming again a Friedmann metric of the form given in equation (4) above, during the inflationary era we have simply

$$\left(\frac{\dot{a}}{a}\right)^2 \approx \frac{1}{L_{inf}^2}, \tag{23}$$

whereas in the pre-inflationary phase we have

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{1}{L_{inf}^2} + \frac{8\pi}{3} \rho_{\text{NRCViolating}}, \tag{24}$$

where $\rho_{\text{NRCViolating}}$ simply measures the extent to which the NRC fails; that is, it measures the disagreement between the NRC and the NEC. Formally, we can think of it as an

"energy density", but it does not necessarily correspond to any actual matter field; nor will it obey the usual energy conditions.

Now string cosmology relies in a fundamental way on T-duality. For example, in the specific case of the Brandenberger-Vafa theory, the expansion of three dimensions away from the initial state is explained by assuming a fluctuation in the form of an event involving the annihilation of winding modes. One could object that the *creation* of winding modes, leading to *contraction*, is just as likely. But T-duality implies that these must simply be two ways of describing the same event. This means that a non-singular string cosmology is *formally* a "bounce" cosmology, but with a crucial difference: the "contracting" phase of the history is just a redundant, dual description of the expanding phase, not an actual physical era.

Thinking about string cosmology in this way makes it clear that the initial state of the Universe must correspond to a spacelike surface with *zero extrinsic curvature*, like the surface of maximal contraction in a true "bounce" cosmology. We see at once that this would not be possible if the strict Einstein equation (23) held in the pre-inflationary era, but it is possible with the modified equation (24), provided that

$$\frac{8\pi}{3} \rho_{\text{NRCViolating}}(0) = \frac{-1}{L_{\text{inf}}^2}, \tag{25}$$

where $t = 0$ at the initial state. That is, the initial total density has to be zero, with the inflaton energy density being exactly cancelled by the negative "energy density" arising from the disagreement between the NEC and the NRC, as discussed above.

Now we know that the NRC has to be violated here, and this means that the total energy density must increase from its initial value, zero. Since $\rho_{\text{NRCViolating}}$ is negative, this just means that its absolute value must *decrease* as the Universe expands away from the initial state. A comparison with the formula for the energy density in the post-inflationary era [equation (6)] suggests the ansatz

$$\rho_{\text{NRCViolating}} = \frac{-3}{8\pi L_{\text{inf}}^2 a^\gamma}, \tag{26}$$

where $\gamma$ is a positive constant and where the coefficients have been chosen so that [equation (25)] the scale factor has unity as its initial value.

Obviously this way of representing the failure of the NRC in the early Universe is far too simple to be realistic. Nevertheless we shall see that it does capture the essential behaviour of the Seiberg-Witten brane action in that era; furthermore it does capture the behaviour of realistic candidates for the origin of NRC violation. For example, if the NRC-violating component is a simple Casimir energy arising from a toral topology [59], we can use the form given in (26) with $\gamma = 4$. The parameter $\gamma$ *is of basic importance here*: it plays the same role in the pre-inflationary era that $\epsilon$ [equation (3)] plays in the post-inflationary era — that is, it fixes the "equation of state". Our main objective now is to understand how it is determined.

With any positive value of $\gamma$, the absolute value of the NRC-violating "energy density", $|\rho_{\text{NRCViolating}}|$, automatically decreases with the expansion, and so the total energy density [the sum of the positive vacuum energy and $\rho_{\text{NRCViolating}}$] *increases*: this means that the NRC is violated overall. The "pressure" corresponding to this negative "energy density"

may be computed from the vanishing of the covariant divergence of the total stress-energy-momentum tensor, that is, from the equation

$$\frac{d}{dt}\rho_{\text{NRCViolating}} + \frac{3\,\dot{a}}{a}\left(\rho_{\text{NRCViolating}} + p_{\text{NRCViolating}}\right) = 0\,; \tag{27}$$

the result is

$$p_{\text{NRCViolating}} = \frac{3}{8\pi L_{\text{inf}}^2\, a^\gamma}\left[1 - \frac{1}{3}\gamma\right]. \tag{28}$$

Combining this with equation (26), we see that the NRC is indeed violated at all times in this geometry: this is consistent with the Andersson-Galloway theorem.

Substituting (26) into (24) and solving, we obtain [60] a family of metrics parametrized by $\gamma$:

$$g_c(\gamma,\, K_0,\, L_{\text{inf}})_{+---} = dt^2 - K_0^2\, \cosh^{(4/\gamma)}\!\left(\frac{\gamma\, t}{2L_{\text{inf}}}\right)[d\theta_1^2 + d\theta_2^2 + d\theta_3^2], \tag{29}$$

where the c subscript refers to the cosh function. Here $K_0$ is simply the initial radius of the spatial torus [which is of course very much smaller than the radius K discussed in previous sections].

There are in fact two ways of thinking about this metric. Taking it at face value, we have here a "bounce" of the kind discussed in, for example, [61]. Such cosmologies have been criticized [see for example [62]] on the grounds that they demand extreme fine-tuning. An alternative approach is to discard the contracting half of the spacetime, and investigate the possibility of "creation from nothing" [7] along $t = 0$. If the initial torus is of the right size [about the string scale] one might also interpret the remaining half of the geometry in terms of string cosmology: by T-duality the "contracting" phase of this geometry merely represents a redundant description of the expanding phase. In order to be definite we shall adopt a "creation from nothing at the string scale" interpretation here: so we are *only* concerned with values of $t \geq 0$ in equation (29).

Since the function $\cosh^{(4/\gamma)}\!\left(\frac{\gamma\, t}{2L_{\text{inf}}}\right)$ is [up to an overall constant factor] asymptotically independent of $\gamma$ and indistinguishable from an exponential function, these spacetimes are "asymptotically STdS", with a toral conformal boundary. Their *past*, however, is very different from that of STdS spacetime: as we hoped, these spacetimes are non-singular, since the scale function never vanishes. We conclude that these geometries are ideally suited to describe the pre-inflationary era: they automatically generate an inflationary geometry in the future, yet they are non-singular at the earliest times. The Penrose diagram [Figure 1] is rectangular, with width[5] $\pi$ and height

$$\Omega(L_{\text{inf}}/K_0,\, \gamma) = \frac{2\, L_{\text{inf}}}{\gamma\, K_0} \int_0^\infty \frac{dx}{\cosh^{(2/\gamma)}(x)}. \tag{30}$$

The horizontal line at the bottom of Figure 1 represents the creation of this universe [presumably from "nothing", as in [9][10][12]] at proper time $t = 0$, which also corresponds to conformal time $\eta = 0$. [That is, this conformal diagram only represents the expanding half of the geometry described by the metric in equation (29).] Now we are interested in fairly *large* values for the ratio $L_{\text{inf}}/K_0$: we shall usually take it to be about 100. It can

---

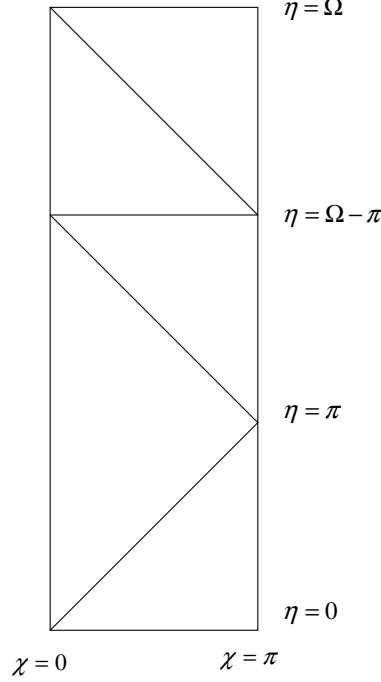[5]Here $\chi$ represents any one of the angular coordinates, which run from $-\pi$ to $+\pi$.

Figure 1: Penrose diagram for the pre-inflationary/inflationary eras.

be shown that, for all values of $\gamma$, this implies that $\Omega(\mathrm{L_{inf}}/\mathrm{K_0}, \gamma)$ is similarly large. Thus, bearing in mind that the Penrose diagram has a width equal to $\pi$, we see that the diagram is very much taller than it is wide. This means that our model gives a natural setting for Linde's [29][30] solution of the problem of getting low-scale Inflation [31] started. Linde observes that it is possible for the Universe to be born with a significantly smaller size than the inflationary scale, and yet still be sufficiently homogeneous for Inflation actually to begin after the Universe expands to the appropriate scale, provided that homogeneity can be maintained by means of chaotic mixing [32] during the pre-inflationary era. This idea was formulated in a more concrete way in [12], as follows.

Let us suppose that the initial spatial section was a torus of side length $2\pi\mathrm{K_0}$, where the radius $\mathrm{K_0}$ is the self-dual scale, that is, the string length scale. The Universe expands in accordance with the geometry given by the metric $g_c(\gamma, \mathrm{K_0}, \mathrm{L_{inf}})_{+---}$, but global causal contact is established provided that the Penrose diagram is *tall enough*. For, in that case, it will be possible for signals to circumnavigate the entire Universe. In Figure 1, the lower

rectangle represents the pre-inflationary era subsequent to creation from "nothing" at t = 0, while the upper square represents the era during which the rate of expansion is so extreme that causal contact begins to be lost: beyond the point $\eta = \Omega - \pi$ in the diagram, the most distant events can no longer affect local physics. The conventional inflationary era must begin some time after this point. As we have just seen, the values of $L_{inf}/K_0$ in which we are interested here ensure that the diagram should in fact be very much taller than it is wide, so there is ample opportunity for a signal to be sent from a point on the torus to the point most distant from it in this direction[6] and back again. [Thus the diagram in Figure 1 is as short as it can be; in reality, it would be much taller.]

From Figure 1 we see that global causal contact begins to be lost, in these directions, $\pi$ units of angular conformal time before the top of the diagram [which represents the end of the era described by this metric, that is, the end of Inflation]. In other directions, global causal contact has already been lost by this time. For Inflation to begin, the Universe must still be sufficiently homogeneous, but we do not need it to be absolutely homogeneous when Inflation begins. That is, we want a short gap between the end of chaotic mixing and the start of inflation.

The geometry we are discussing here ensures this automatically, at least for values of $\gamma$ that are not too small. To see this, let $T_1$ be the proper time [in equation (29)] when global causal contact begins to break down: this is given by

$$\pi \;=\; \frac{2\,L_{inf}}{\gamma\,K_0} \int_{(\gamma T_1/2L_{inf})}^{\infty} \frac{dx}{\cosh^{(2/\gamma)}(x)} \,. \tag{31}$$

Next, let $T_2$ be the proper time at which Inflation begins, that is, the time at which the string scale [given by $K_0$] was stretched to the inflationary scale [given by $L_{inf}$]. We have then

$$\frac{L_{inf}}{K_0} \;=\; \cosh^{(2/\gamma)}\Big(\frac{\gamma T_2}{2L_{inf}}\Big). \tag{32}$$

Judicious use of the elementary inequalities $\frac{1}{2}e^x < \cosh(x) \le e^x$, where $x \ge 0$, allows us to combine these relations to obtain

$$\frac{T_2}{L_{inf}} \;>\; \frac{T_1}{L_{inf}} \;+\; \ln(\pi/2^{(2/\gamma)}). \tag{33}$$

The second term on the right is positive provided that $\gamma$ exceeds about 1.21, as will be the case for all of the values we shall discuss below. Thus Inflation does indeed begin after chaotic mixing fails. The gap between the two events depends on $\gamma$. In principle one could constrain $\gamma$ by imposing conditions on the amount of inhomogeneity produced during the gap, but the present model is too simplified for further investigation to be worth while here; however, it might be of interest in more complex models.

Thus we have a simple model of the pre-inflationary era, one which leads to a spacetime structure reflecting the basic physical requirements: it is non-singular and is able to accommodate a theory of initial conditions for Inflation. We claim that a realistic theory of the earliest Universe would not lead to a spacetime geometry vastly different from this

---

[6]On a torus, the distance to the "most distant point" depends on direction. In practice our Penrose diagrams will be so tall that this will not be an issue.

one. It is therefore appropriate to investigate the stability of this spacetime, in the sense of Seiberg and Witten. The relevant family of Euclidean metrics here is given by

$$g_c(\gamma, \, K_0, \, L_{inf})_{++++} \; = \; K_0^2 \cosh^{(4/\gamma)}\Big(\frac{\gamma \, u(\eta_+)}{2 \, L_{inf}}\Big) \, [\, d\eta_+^2 \; + \; d\theta_1^2 \; + \; d\theta_2^2 \; + \; d\theta_3^2], \qquad (34)$$

where as usual $\eta_+$ is Euclidean conformal time; we take it that $\eta_+$ runs from $-\,\Omega(L_{inf}/K_0, \, \gamma)$ [see equation (30); this corresponds to the smallest transverse section, the Euclidean version of the initial spatial section], to zero [which represents the conformal boundary, as with the Euclidean conformal time coordinate of STdS itself — see equation (13)]. The function $u(\eta_+)$ vanishes at $\eta_+ \; = \; -\,\Omega(L_{inf}/K_0, \, \gamma)$ and satisfies

$$\frac{du}{d\eta_+} \; = \; K_0 \cosh^{(2/\gamma)}\Big(\frac{\gamma \, u(\eta_+)}{2 \, L_{inf}}\Big). \qquad (35)$$

The Seiberg-Witten action for a brane of tension T created at the smallest transverse section is

$$S(g_c(\gamma, \, K_0, \, L_{inf})_{++++} \, ; \, \eta_+) \; = \quad 8\pi^3 \, K_0^3 \, T\Big[\cosh^{(6/\gamma)}\Big(\frac{\gamma \, u(\eta_+)}{2L_{inf}}\Big)$$
$$-\; \frac{3}{L_{inf}} \int_0^u \cosh^{(6/\gamma)}\Big(\frac{\gamma \, u}{2L_{inf}}\Big) \, du\Big]. \qquad (36)$$

Since $u = 0$ at the initial point, the action is initially positive: the initial value is just $8\pi^3 K_0^3 \, T$. Unlike the action in the post-inflationary era [equation (18)], however, this action *decreases* steadily as we move towards infinity[7]. *It is therefore far from clear that it will remain positive.*

There are values of $\gamma$ which *certainly do* lead to Seiberg-Witten instability if the metric continues to have the form (29): for example, with $\gamma = 3$ one has

$$S(g_c(3, \, K_0, \, L_{inf})_{++++} \, ; \, \eta_+) \; = \; 4\pi^3 K_0^3 \, T[1 \; + \; e^{-\, 3u(\eta_+)/L_{inf}} \; - \; \frac{3u(\eta_+)}{L_{inf}}], \qquad (37)$$

which becomes negative and is in fact unbounded below. However, we shall see later that, for other values of $\gamma$, it is possible for the action to decrease so slowly that it never becomes negative. Thus the situation in the pre-inflationary era is much more complex than in the post-inflationary era. In the latter case, stability is guaranteed, for *all* values of $\epsilon$, provided of course that the brane action is positive at the end of Inflation. Here, by contrast, we find that stability is not guaranteed: it depends on the value of $\gamma$.

In fact, we can turn this argument around, and use the requirement of stability to constrain $\gamma$. Once $\gamma$ is fixed, the pre-inflationary geometry is likewise fixed. We now turn to this question.

## 5. Fixing the Pre-Inflationary Spacetime Geometry

We saw in the previous section that certain values of $\gamma$ lead to a particular form of instability, while others do not. We shall now argue that this is just one of several constraints on $\gamma$; though in fact it will prove to be the strongest.

---

[7]Note that equation (35) implies that $\lim_{\eta_+ \to 0} u(\eta_+) = \infty$.

We begin with the following elementary observation. If we assemble the "energy density" $\rho_{\text{NRCViolating}}$ and the "pressure" $p_{\text{NRCViolating}}$ into a four-vector, this vector will be timelike or null precisely when $\gamma \leq 6$, as can be seen by comparing equations (26) and (28). Values of $\gamma$ greater than 6 correspond to an "energy-momentum vector" which is *spacelike*. Now we should not immediately conclude that causality will be violated in this case: a cosmological "equation of state" is not a true equation of state from which a "speed of sound" can be deduced. [For example, the speed of signal propagation in a quintessence field is always locally equal to the speed of light, whatever the "equation of state" may be [63]. See [64] for a general discussion of cosmological equations of state.] Even granting this, however, and even granting that we do not necessarily interpret $\rho_{\text{NRCViolating}}$ and $p_{\text{NRCViolating}}$ in a literal way as energy density and pressure, the presence of a spacelike vector in this context is unwelcome[8]; one should view such an object with suspicion unless one has physical arguments to the contrary [see for example [65]].

These suspicions are strongly supported by the fact that it can be shown [12] that the action of the Euclidean instanton defined by $g_c(\gamma, \text{K}_0, \text{L}_{\text{inf}})_{+---}$ diverges precisely when $\gamma > 6$. Such values are therefore ruled out by *any* version of Euclidean quantum gravity. This is the case, for example, whether one uses the Hartle-Hawking wave function or some modification of it [9]. We stress that, because of this last remark, this argument is much more general than the final conclusions reached in [12], which do depend on the details of the wave function.

In short, classical arguments and simple but general quantum-gravitational considerations lead us to focus on values of $\gamma$ which are less than or equal to 6.

In considering such values, we must bear in mind the fact that the brane action given in (36) is *initially* positive; it can only become negative at some point deeper into the space. Now the metric (29) is only supposed to describe the geometry until the late inflationary era; for example, with a conventional scalar field inflaton, when the kinetic term in the inflaton lagrangian ceases to be negligible, and radiation begins to become important, we must switch to a completely different metric. That metric will in fact be approximated, at first, by the metric $g_s(4, \text{K}, \text{L})_{+---}$ [equation (5)], where K measures the size of the torus at the end of Inflation, where L is the length scale determined by the *current* value of the cosmological constant, and where we have set $\epsilon = 4$ to describe radiation. But we know that the brane action in the Euclidean version of this geometry will actually *increase* [equation (20)], and that this will continue to be the case when matter begins to dominate over radiation [that is, when $\epsilon = 4$ is eventually replaced by $\epsilon = 3$]. In other words, the brane action will actually cease to decrease at some point. It is conceivable that, if the action has been decreasing sufficiently slowly up to that point, *it may never have become negative even if the expression given in (36) indicates that it would ultimately do so if Inflation never ended.* That is, what we should require is that the action should remain positive up to this point, not necessarily everywhere.

Take for example the case of $\gamma = 3$. Clearly [equation (37)] the action in this case *can* become negative; in fact, a numerical investigation shows that it becomes negative at a value of $u(\eta_+)$ given approximately by $0.4262 \, \text{L}_{\text{inf}}$. But a further numerical investigation using equation (31) shows that, assuming $\text{L}_{\text{inf}} \approx 100 \, \text{K}_0$, the pre-Inflationary era ends at

---

[8]Note that this argument does not apply to the *total* energy density and pressure, because dark energy in the form of a cosmological constant cannot "carry a signal".

a time corresponding to around $u(\eta_+) = 3.923\,L_{inf}$ for $\gamma = 3$. Thus, in this case, it is clear that a severe instability will develop during the period when the metric (29) is still valid — that is, long before the end of Inflation. In fact, in this particular case, the instability develops even before Inflation begins. Thus $\gamma = 3$ is certainly ruled out.

The case $\gamma = 4$ is of particular interest, since that is the value to be expected if the NRC-violating component is associated with the Casimir effect [59]. A numerical investigation of (36) in this case shows that, in this case, the action becomes negative at $u(\eta_+)$ equal to about $0.4947\,L_{inf}$, but (31) implies, again assuming $L_{inf} \approx 100\,K_0$, that the pre-Inflationary era ends at around $3.8070\,L_{inf}$; thus, once again, the system becomes unstable during the pre-inflationary era itself. These conclusions do not change significantly if we take $L_{inf} \approx 10\,K_0$ — the pre-inflationary era then ends at about $1.5042\,L_{inf}$, but this is still well beyond the point where the action becomes negative. [If we take $L_{inf} \approx 1000\,K_0$, then the pre-inflationary era ends at about $6.1096\,L_{inf}$.] Thus it seems that Casimir effects cannot account for the non-singularity of string cosmology. This is an important and unexpected conclusion.

| $\gamma$ | $S = 0$ | $T_1$ | $T_2$ |
|---|---|---|---|
| 5.0000 | 0.6346 | 3.7377 | 4.8824 |
| 5.9000 | 1.2539 | 3.6954 | 4.8401 |
| 5.9900 | 1.9882 | 3.6919 | 4.8366 |
| 5.9990 | 7.5654 | 3.6915 | 4.8363 |
| 6.0000 | $\infty$ | 3.6915 | 4.8362 |

It turns out that the brane action vanishes at a point which always recedes towards infinity as $\gamma$ increases. See the Table; here $T_1$ and $T_2$ are as in equations (31) and (32), that is, they represent respectively the end of the pre-inflationary era and the start of Inflation. [$L_{inf} \approx 100\,K_0$ is assumed throughout, and the units are given by the inflationary length scale.] The rate at which the zero point moves towards infinity is very slow at first; the zero point is still within the region corresponding to the pre-inflationary era even at $\gamma = 5.99$. But the zero point begins to move towards infinity at a rate which gathers pace dramatically as $\gamma$ nears the value 6. Increasing $\gamma$ only slightly, from $\gamma = 5.99$ to $\gamma = 5.999$, causes the zero point to jump well into the region corresponding to the inflationary era. But this is of course still unacceptable: we want the action to be positive throughout that region, that is, for at least 60 e-folds from the point $u(\eta_+) = T_2$. It is clear that this can be done, but it is also clear that this will require $\gamma$ to be extremely close to 6.

For $\gamma = 6$ itself, the action can be evaluated explicitly: it is given by

$$S(g_c(6,\, K_0,\, L_{inf})_{++++}\,;\, \eta_+) \;=\; 8\pi^3 K_0^3\, T\, e^{(-\,3u(\eta_+)/L_{inf})}, \tag{38}$$

which is manifestly positive everywhere.

We saw earlier that there are strong physical arguments suggesting that $\gamma$ can be no *larger* than 6. Here we have seen that brane physics demands that $\gamma$ can be only very slightly *smaller* than 6. Thus $\gamma$ is now tightly constrained. We conclude that the spacetime geometry of the earliest Universe can, within the simple model we are considering here, be approximated by the metric

$$g_c(6,\, K_0,\, L_{inf})_{+---} \;=\; dt^2 \;-\; K_0^2\, \cosh^{(2/3)}\!\Big(\frac{3\,t}{L_{inf}}\Big)\,[d\theta_1^2 \;+\; d\theta_2^2 \;+\; d\theta_3^2], \tag{39}$$

defined on a manifold of topology $\mathbb{R} \times \mathrm{T}^3$. Here $\mathrm{K}_0$ is the string scale and $\mathrm{L}_{\mathrm{inf}}$ is the inflationary scale.

The final picture is as follows. With $\gamma = 6$, we have a brane action which decreases from its initial positive value. The decrease continues throughout the regions of the Euclidean space corresponding to the pre-inflationary and inflationary eras. By the point corresponding to end of Inflation, the action is extremely small [equation (38)], *but it is still positive.* Beyond that point, the action begins to increase, and it continues to increase [equation (20)] indefinitely as we move towards the boundary at infinity. Thus the action is positive everywhere; there is no Seiberg-Witten instability in this system.

In [12] we attempted to use the Firouzjahi-Sarangi-Tye wave function [9][10] to predict the *most probable* value of $\gamma$, assuming that a spatially toral Universe could be created from "nothing" with any value of $\gamma$ greater than[9] 3. We found that, by an overwhelming factor, the most probable value was precisely $\gamma = 6$. Thus, these two very different arguments agree. The argument based on the FST wave function actually refines the conclusion slightly, because it turns out that $\gamma = 6$ is very much more probable than any lower value, *no matter how close* it may be to 6: this happens because of a peculiar discontinuity in the wave function at $\gamma = 6$.

One important question remains: what is the physical nature of the NRC-violating field we have been using here? We have seen that it is *not* a Casimir component, but what are the alternatives? Inserting $\gamma = 6$ into equations (26) and (28), we see that the pressure of the NRC-violating component is negative and equal to the density; both density and pressure decay rapidly with the expansion, according to the sixth power of the scale factor. We should attempt to identify the kinds of fields that can behave in this way.

One possibility was discussed recently by Patil [66], who points out that such densities and pressures can be produced by a scalar field with a reversed kinetic term and a *vanishing* potential. The absence of any potential is certainly very interesting in terms of string physics. However, here we are not trying to use such densities and pressures to replace the inflaton, only to supplement it in such a way that an initial singularity is avoided. Furthermore the question of stability is always difficult when this procedure is used.

An alternative that is more consistent with our approach here appears in the work of Gabadadze and Shang [34]. As mentioned earlier, the non-dynamical constraint field in "classically constrained" gravity can lead to apparently negative "energy densities" in a physically acceptable manner. In fact, the Friedmann equation in this case [34] takes the form [with flat sections]

$$\left(\frac{\dot{\mathrm{a}}}{\mathrm{a}}\right)^2 = \frac{1}{\mathrm{L}_{\mathrm{inf}}^2} - \frac{\mathrm{b}\,\varepsilon}{6\,\mathrm{a}^6}\,, \tag{40}$$

where $\mathrm{b}\varepsilon$ is a certain constant which may be positive or negative. In the positive case, with a suitable choice of $\mathrm{b}\varepsilon$, *this is precisely* the equation (24) which we solved to obtain our metric (39): the sixth power is just what we need.

Thus we suggest tentatively that the NRC-violating structure in the pre-inflationary

---

[9]This restriction was based on the assumption, confirmed here, that Inflation would never get started if $\gamma$ were smaller than or equal to 3.

era is just the Gabadadze-Shang constraint field. The spatial sections are [compact] manifolds-with-boundary in [34], not tori; however, it should not be difficult to reconcile these two views, since both spaces are flat and compact. This appears to be a promising way of understanding the physics of NRC violation — and therefore of singularity avoidance — in the early Universe.

## 6. Conclusion

The Andersson-Galloway theorems [see the Appendix] mean that it is very difficult for an accelerating toral cosmology to be non-singular. This is not a drawback: it merely implies that the geometry of the earliest Universe must be strongly constrained. Here we have tried to assess just how strong those constraints may be in the context of a particular framework for string cosmology, one based on Linde's suggestion that chaotic mixing may have a crucial role to play in the earliest stages of Inflation.

What we find is that, with a few simple assumptions, the spacetime geometry is completely fixed. The reader may wish to argue that the model we have considered, based on the simple ansatz given in equation (26), is in fact too simple — though such simplifications are standard practice in cosmology, and indeed (26) is based directly on the familiar equation (6). However, in [28] we studied the more complex explicit NRC-violating spacetimes constructed by Aref'eva et al [67]; using the analysis developed there, one can show that these spacetimes exhibit essentially the same behaviour as the ones studied here. The general conclusion is that a combination of very simple, *essentially geometric* requirements [formal energy-momentum vectors should not be spacelike, actions of Euclidean instantons should not diverge, the Seiberg-Witten brane action should not become negative] suffice to impose surprisingly strong constraints on the pre-inflationary spacetime geometry. The constraints are indeed strong enough to allow us to specify the spacetime metric, and thus to suggest a candidate for the origin of NRC violation: the Gabadadze-Shang constraint field [34].

It is striking that such strong conclusions flow from the basic idea that the spatial sections of our Universe may have a non-trivial topology. We have claimed that compact, flat manifolds are best suited to quantum cosmology, but other compact candidates [68][69][34][70] certainly merit further attention.

## Appendix: About The Andersson-Galloway Theorem

In this appendix we briefly explain the terminology used by Andersson and Galloway [21][22], and comment on the conditions assumed in their theorem used above. This is

important, because our argument is based on the claim that there is only one way to circumvent the theorem — to violate the Null Ricci Condition.

A four-dimensional spacetime $M_4$ with Lorentzian metric $g_M$ is said to have a *regular future spacelike conformal boundary* if $M_4$ can be regarded as the interior of a spacetime-with-boundary $X_4$, with a [non-degenerate] metric $g_X$ such that the boundary is *spacelike* and lies to the future of all points in $M_4$, while $g_X$ is conformal to $g_M$, that is, $g_X = \Omega^2 g_M$, where $\Omega = 0$ along the boundary but $d\Omega \neq 0$ there. This is just a technical formulation of the idea that the spacetime should be generic and de Sitter-like at late times. There are examples of accelerating spacetimes which do not have a regular future spacelike conformal boundary, but these either severely violate the NRC at very *late* times [45], thus apparently involving massive violations of the NEC over long periods of time, or are highly non-generic, like Nariai spacetime [68] [which is on the very brink of having a naked singularity]. One certainly should not hope to escape from the conclusions of the Andersson-Galloway theorem by resorting to such examples.

A spacetime is said to be *globally hyperbolic* if it possesses a Cauchy surface, that is, a surface on which data can be prescribed which determine all physical fields at later and earlier events in spacetime, since all inextensible timelike and null curves intersect it. This forbids naked singularities, but it also disallows ordinary AdS. The dependence of the Andersson-Galloway theorem on this assumption might lead one to ask whether our conclusions can be circumvented by dropping global hyperbolicity. This is a topical suggestion, since it has recently been claimed [71] that string theory allows spacetimes which violate global hyperbolicity even more drastically than AdS. To see why this too will not work here, we need the following definition.

A spacetime with a regular future spacelike conformal completion is said to be *future asymptotically simple* if every future inextensible null geodesic has an endpoint on future conformal infinity. This just means that there are no singularities to the future — obviously a reasonable condition to impose in our case, since it would be bizarre to suppose that singularities to the future can somehow allow us to avoid a Big Bang singularity. Andersson and Galloway [21] show, however, that if a spacetime has a regular future spacelike conformal completion and is future asymptotically simple, *then it has to be globally hyperbolic*. Thus it would not be reasonable to drop this condition in our context. Notice that this discussion has a more general application: it means that, if the future of our Universe resembles that of de Sitter spacetime, any attempt to violate global hyperbolicity will necessarily cause a singularity to develop to the future. It would be interesting to understand this in the context of [71].

Finally, one might wonder whether a compact flat three-manifold can in fact have a first homology group which is pure torsion. The rather surprising answer is that it can: there is a unique manifold of this kind, the *didicosm*, described in [72][1]. The didicosm has the form $T^3/[\mathbb{Z}_2 \times \mathbb{Z}_2]$, that is, it is a quotient of the three-torus. However, if it were possible to construct a singularity-free spacetime metric on $\mathbb{R} \times T^3/[\mathbb{Z}_2 \times \mathbb{Z}_2]$ without violating the NRC, this metric would pull back, via an obvious extension of the covering map $T^3 \to T^3/[\mathbb{Z}_2 \times \mathbb{Z}_2]$, to a non-singular metric on $\mathbb{R} \times T^3$, also satisfying the NRC. This is a contradiction. Similarly, of course, one cannot escape the conclusions of the theorem by allowing spacelike future infinity to be non-orientable or by using orbifolds.

We conclude that the only physically reasonable way to avoid a Bang singularity in

an accelerating cosmology of toral spatial topology is indeed to violate the NRC.

# References

[1] Brett McInnes, Inflation, Large Branes, and the Shape of Space, Nucl. Phys. B709 (2005) 213, arXiv:hep-th/0410115

[2] Richard Easther, Brian R. Greene, Mark G. Jackson, Cosmological String Gas on Orbifolds, Phys.Rev. D66 (2002) 023502, arXiv:hep-th/0204099

[3] D. Lust, S. Reffert, W. Schulgin, S. Stieberger, Moduli Stabilization in Type IIB Orientifolds (I): Orbifold Limits, arXiv:hep-th/0506090

[4] Hirosi Ooguri, Cumrun Vafa, Erik Verlinde, Hartle-Hawking Wave-Function for Flux Compactifications: The Entropic Principle, Lett.Math.Phys. 74 (2005) 311, arXiv:hep-th/0502211

[5] Robbert Dijkgraaf, Rajesh Gopakumar, Hirosi Ooguri, Cumrun Vafa, arXiv:hep-th/0504221; Sergei Gukov, Kirill Saraikin, Cumrun Vafa, arXiv:hep-th/0505204; Sergei Gukov, Kirill Saraikin, Cumrun Vafa, arXiv:hep-th/0509109; Bartomeu Fiol, arXiv:hep-th/0602103

[6] J.B. Hartle and S.W. Hawking, Wave Function of the Universe, Phys.Rev.D28 (1983) 2960

[7] Ya.B.Zel'dovich, A.A. Starobinsky, Quantum Creation of a Universe with Nontrivial Topology, Pis'ma Astron. Zh. 10 (1984) 323

[8] Jorma Louko, Peter J. Ruback, Spatially flat quantum cosmology, Class.Quant.Grav. 8 (1991) 91

[9] Hassan Firouzjahi, Saswat Sarangi, S.-H. Henry Tye, Spontaneous Creation of Inflationary Universes and the Cosmic Landscape, JHEP 0409 (2004) 060, arXiv:hep-th/0406107

[10] Saswat Sarangi, S.-H. Henry Tye, The Boundedness of Euclidean Gravity and the Wavefunction of the Universe, arXiv:hep-th/0505104

[11] Saswat Sarangi, S.-H. Henry Tye, A Note on the Quantum Creation of Universes, arXiv:hep-th/0603237

[12] Brett McInnes, The Most Probable Size of the Universe, Nucl.Phys. B730 (2005) 50, arXiv:hep-th/0509035

[13] Archil Kobakhidze, Laura Mersini-Houghton, arXiv:hep-th/0410213; Laura Mersini-Houghton, Class.Quant.Grav. 22 (2005) 3481, arXiv:hep-th/0504026; Qing-Guo Huang, arXiv:hep-th/0510219; Qing-Guo Huang, arXiv:hep-th/0512004

[14] R. H. Brandenberger and C.Vafa, Nucl.Phys.B 316 (1989) 391; S. Alexander, R. Brandenberger, D. Easson, Phys.Rev. D62 (2000) 103509, arXiv:hep-th/0005212; Damien A. Easson, Int.J.Mod.Phys. A18 (2003) 4295, arXiv:hep-th/0110225; Robert Brandenberger, Damien A. Easson, Dagny Kimberly, Nucl.Phys. B623 (2002) 421, arXiv:hep-th/0109165; Robert Brandenberger, Damien A. Easson, Anupam Mazumdar, Phys.Rev. D69 (2004) 083502, arXiv:hep-th/0307043; Tirthabir Biswas, Robert Brandenberger, Damien A. Easson, Anupam Mazumdar, Phys.Rev. D71 (2005) 083514, arXiv:hep-th/0501194; Damien A. Easson, Mark Trodden, Phys.Rev. D72 (2005) 026002, arXiv:hep-th/0505098; Aaron Berndsen, Tirthabir Biswas, James M. Cline, JCAP 0508 (2005) 012, arXiv:hep-th/0505151; Paul Robert Chouha, Robert H. Brandenberger, arXiv:hep-th/0508119; Robert H. Brandenberger, arXiv:hep-th/0509076, arXiv:hep-th/0509099, arXiv:hep-th/0509159; Francesc Ferrer, Syksy Rasanen, JHEP 0602 (2006) 016, arXiv:hep-th/0509225

[15] Thorsten Battefeld, Scott Watson, String Gas Cosmology, arXiv:hep-th/0510022

[16] John McGreevy, Eva Silverstein, The Tachyon at the End of the Universe, JHEP 0508 (2005) 090, arXiv:hep-th/0506130

[17] D. J. Eisenstein et al, Detection of the Baryon Acoustic Peak in the Large-Scale Correlation Function of SDSS Luminous Red Galaxies, Astrophys.J. 633 (2005) 560, arXiv:astro-ph/0501171

[18] Kazuhide Ichikawa, Tomo Takahashi, Dark Energy Evolution and the Curvature of the Universe from Recent Observations, arXiv:astro-ph/0511821

[19] Dylan Menzies, Grant J. Mathews, An Axisymmetric Object-Based Search for a Hypertorus Cosmology, JCAP 0510 (2005) 008, arXiv:astro-ph/0503555

[20] M.J. Reboucas, J.S. Alcaniz, B. Mota, M. Makler, Supernovae observations and cosmic topology, arXiv:astro-ph/0511007

[21] L. Andersson, G.J. Galloway, dS/CFT and spacetime topology, Adv.Theor.Math.Phys. 6 (2003) 307, arXiv:hep-th/0202161

[22] Gregory J. Galloway, Cosmological Spacetimes with $\Lambda > 0$, arXiv:gr-qc/0407100

[23] Nathan Seiberg, Edward Witten, The D1/D5 System And Singular CFT, JHEP 9904 (1999) 017, arXiv:hep-th/9903224

[24] Brett McInnes, Stringy Instability of Topologically Non-Trivial AdS Black Holes and of Asymptotically deSitter S-brane Spacetimes, Nucl. Phys. B660 (2003) 373, arXiv:hep-th/0205103

[25] Juan Maldacena, Liat Maoz, Wormholes in AdS, JHEP 0402 (2004) 053, arXiv:hep-th/0401024

[26] Alex Buchel, Phys.Rev. D70 (2004) 066004, arXiv:hep-th/0402174; Brett McInnes, JHEP 0404 (2004) 036, arXiv:hep-th/0403104; Brett McInnes, JHEP 10(2004)018, arXiv:hep-th/0407189; Jan de Boer, Liat Maoz, Asad Naqvi, arXiv:hep-th/0407212;

E. Bergshoeff, A. Collinucci, A. Ploegh, S. Vandoren, T. Van Riet, JHEP 0601 (2006) 061, arXiv:hep-th/0510048

[27] M. Kleban, M. Porrati, R. Rabadan, Stability in Asymptotically AdS Spaces, JHEP 0508 (2005) 016, arXiv:hep-th/0409242

[28] Brett McInnes, The Phantom Divide in String Gas Cosmology, Nucl.Phys. B718 (2005) 55, arXiv:hep-th/0502209

[29] Andrei Linde, Creation of a Compact Topologically Nontrivial Inflationary Universe, JCAP 0410 (2004) 004, arXiv:hep-th/0408164

[30] Andrei Linde, Inflation and String Cosmology, eConf C040802 (2004) L024; J.Phys.Conf.Ser. 24 (2005) 151, arXiv:hep-th/0503195

[31] J.J. Blanco-Pillado, C.P. Burgess, J.M. Cline, C. Escoda, M. Gomez-Reino, R. Kallosh, A. Linde, F. Quevedo, Racetrack Inflation, JHEP 0411 (2004) 063, arXiv:hep-th/0406230

[32] Neil J. Cornish, David N. Spergel, Glenn D. Starkman, Does Chaotic Mixing Facilitate $\Omega < 1$ Inflation?, Phys.Rev.Lett. 77 (1996) 215, arXiv:astro-ph/9601034

[33] Gregory Gabadadze, Yanwen Shang, Classically Constrained Gauge Fields and Gravity, arXiv:hep-th/0506040

[34] Gregory Gabadadze, Yanwen Shang, Quantum Cosmology of Classically Constrained Gravity, arXiv:hep-th/0511137

[35] R. Brustein, S. P. de Alwis, The Landscape of String Theory and The Wave Function of the Universe, Phys.Rev. D73 (2006) 046009, arXiv:hep-th/0511093

[36] R. Holman, L. Mersini-Houghton, A Fly in the SOUP, arXiv:hep-th/0511112

[37] Alexander Friedland, Hitoshi Murayama, Maxim Perelstein, Domain Walls as Dark Energy, Phys.Rev. D67 (2003) 043519, arXiv:astro-ph/0205520

[38] R. J. Adler, J. M. Overduin, The nearly flat universe, Gen.Rel.Grav. 37 (2005) 1491, arXiv:gr-qc/0501061

[39] E. Babichev, V. Dokuchaev, Yu. Eroshenko, Dark energy cosmology with generalized linear equation of state, Class.Quant.Grav. 22 (2005) 143, arXiv:astro-ph/0407190

[40] Adam G. Riess et al, Type Ia Supernova Discoveries at z > 1 From the Hubble Space Telescope: Evidence for Past Deceleration and Constraints on Dark Energy Evolution, Astrophys.J. 607 (2004) 665, arXiv:astro-ph/0402512

[41] T. Banks, L. Mannelli, W. Fischler, Infrared Divergences in dS/CFT, arXiv:hep-th/0507055

[42] Arvind Borde, Alex Vilenkin, Phys.Rev. D56 (1997) 717, arXiv:gr-qc/9702019; Arvind Borde, Alan H. Guth, Alexander Vilenkin, Phys.Rev.Lett. 90 (2003) 151301, arXiv:gr-qc/0110012

[43] Brett McInnes, The Covariant Entropy Bound, Brane Cosmology, and the Null Energy Condition, JHEP 0212 (2002) 053, arXiv:hep-th/0212014

[44] Alessandro Melchiorri, Laura Mersini, Carolina J. Odman, Mark Trodden, Phys.Rev. D68 (2003) 043509, arXiv:astro-ph/0211522; Sean M. Carroll, Mark Hoffman, Mark Trodden, Phys.Rev. D68 (2003) 023509, arXiv:astro-ph/0301273; Roman V. Buniy, Stephen D.H. Hsu, Phys.Lett. B632 (2006) 543, arXiv:hep-th/0502203; A. Coley, S. Hervik, J. Latta, arXiv:astro-ph/0503169; L. Raul Abramo, Nelson Pinto-Neto, arXiv:astro-ph/0511562; S. Dubovsky, T. Gregoire, A. Nicolis, R. Rattazzi, arXiv:hep-th/0512260

[45] R.R. Caldwell, Phys.Lett. B545 (2002) 23, arXiv:astro-ph/9908168; Brett McInnes, JHEP 08 (2002) 029, arXiv:hep-th/0112066; V.K. Onemli, R.P. Woodard, Class.Quant.Grav. 19 (2002) 4607, arXiv:gr-qc/0204065. For the most recent developments see: Francisco S. N. Lobo, Phys.Rev. D71 (2005) 124022, arXiv:gr-qc/0506001; E.Babichev, V.Dokuchaev, Y.Eroshenko, arXiv:gr-qc/0507119; Massimo Giovannini, Phys.Rev. D72 (2005) 083508, arXiv:astro-ph/0507369; I. Brevik, O. Gorbunova, Y. A. Shaido, Int.J.Mod.Phys. D14 (2005) 1899, arXiv:gr-qc/0508038; Tirthabir Biswas, Anupam Mazumdar, Warren Siegel, arXiv:hep-th/0508194; Pu-xun Wu, Hong-wei Yu, arXiv:gr-qc/0509036; Marek Szydlowski, arXiv:astro-ph/0509415; Jose A. Jimenez Madrid, Pedro F. Gonzalez-Diaz, arXiv:astro-ph/0510051; Sushil K. Srivastava, arXiv:astro-ph/0511167; Hrvoje Stefancic, arXiv:astro-ph/0511316; Songbai Chen, Jiliang Jing, arXiv:gr-qc/0511085; Valerio Faraoni, Shahn Nadeau, arXiv:gr-qc/0511094; Jun-Qing Xia, Gong-Bo Zhao, Bo Feng, Hong Li, Xinmin Zhang, arXiv:astro-ph/0511625; Jose A. Jimenez Madrid, arXiv:astro-ph/0512117; A.A. Coley, S. Hervik, W.C. Lim, arXiv:gr-qc/0512085; Jailson Alcaniz, H. Stefancic, arXiv:astro-ph/0512622; Leandros Perivolaropoulos, arXiv:astro-ph/0601014; Guido Cognola, Emilio Elizalde, Shin'ichi Nojiri, Sergei D. Odintsov, Sergio Zerbini, arXiv:hep-th/0601008; Iver Brevik, arXiv:gr-qc/0601100; Shin'ichi Nojiri, Sergei D. Odintsov, arXiv:hep-th/0601213; Ishwaree P. Neupane, arXiv:hep-th/0602097; Sushil K. Srivastava, arXiv:astro-ph/0602116; Valerio Faraoni, Michael N. Jensen, arXiv:gr-qc/0602097; Iver Brevik, arXiv:gr-qc/0603025; Edmund J. Copeland, M. Sami, Shinji Tsujikawa, arXiv:hep-th/0603057; Shin'ichi Nojiri, Sergei D. Odintsov, arXiv:hep-th/0603062

[46] Csaba Csaki, Nemanja Kaloper, John Terning, The Accelerated Acceleration of the Universe, arXiv:astro-ph/0507148

[47] Subinoy Das, Pier Stefano Corasaniti, Justin Khoury, Super-acceleration as Signature of Dark Sector Interaction, arXiv:astro-ph/0510628

[48] Manoj Kaplinghat, Arvind Rajaraman, Stable Models of Super-acceleration, arXiv:astro-ph/0601517

[49] Martin Schaden, arXiv:hep-th/0509124; Carlos A. R. Herdeiro, Marco Sampaio, Class.Quant.Grav. 23 (2006) 473, arXiv:hep-th/0510052; N. Ahmadi, M. Nouri-Zonoz, Nucl.Phys. B738 (2006) 269, arXiv:gr-qc/0510100

[50] Varun Sahni, arXiv:astro-ph/0502032; Ujjaini Alam, Varun Sahni, arXiv:astro-ph/0511473; A. A. Coley, arXiv:astro-ph/0504226; G. Kofinas, G. Panotopoulos, T.N. Tomaras, JHEP 0601 (2006) 107, arXiv:hep-th/0510207

[51] Emilio Elizalde, Shin'ichi Nojiri, Sergei D. Odintsov, Peng Wang, Dark Energy: Vacuum Fluctuations, the Effective Phantom Phase, and Holography, Phys.Rev. D71 (2005) 103504, arXiv:hep-th/0502082

[52] Pantelis S. Apostolopoulos, Nikolaos Tetradis, Brane cosmological evolution with a general bulk matter configuration, Phys.Rev. D71 (2005) 043506, arXiv:hep-th/0412246

[53] Pantelis S. Apostolopoulos, Nikolaos Tetradis, The Generalized Dark Radiation and Accelerated Expansion in Brane Cosmology, Phys.Lett. B633 (2006) 409, arXiv:hep-th/0509182

[54] Shin'ichi Nojiri, Sergei D. Odintsov, Misao Sasaki, Gauss-Bonnet dark energy, Phys.Rev. D71 (2005) 123509, arXiv:hep-th/0504052

[55] Shin'ichi Nojiri, Sergei D. Odintsov, O.G. Gorbunova, Dark energy problem: from phantom theory to modified Gauss-Bonnet gravity, arXiv:hep-th/0510183

[56] B. Boisseau, G. Esposito-Farese, D. Polarski, A.A. Starobinsky, Reconstruction of a scalar-tensor theory of gravity in an accelerating universe, Phys.Rev.Lett. 85 (2000) 2236, arXiv:gr-qc/0001066

[57] Jerome Martin, Carlo Schimd, Jean-Philippe Uzan, Testing for w < −1 in the Solar System, arXiv:astro-ph/0510208

[58] S. Kachru, L. McAllister, Bouncing Brane Cosmologies from Warped String Compactifications, JHEP 0303 (2003) 018, arXiv:hep-th/0205209

[59] D.H. Coule, Quantum Cosmological Models, Class.Quant.Grav. 22 (2005) R125, arXiv:gr-qc/0412026

[60] Brett McInnes, The dS/CFT Correspondence and the Big Smash, JHEP 08 (2002) 029, arXiv:hep-th/0112066

[61] G. Veneziano, A Model for the Big Bounce, JCAP 0403 (2004) 004, arXiv:hep-th/0312182

[62] S.W. Hawking, Thomas Hertog, Populating the Landscape: A Top Down Approach, arXiv:hep-th/0602091

[63] P. J. E. Peebles, Bharat Ratra, The Cosmological Constant and Dark Energy, Rev.Mod.Phys. 75 (2003) 559, arXiv:astro-ph/0207347

[64] Shin'ichi Nojiri, Sergei D. Odintsov, Inhomogeneous Equation of State of the Universe: Phantom Era, Future Singularity and Crossing the Phantom Barrier, Phys.Rev. D72 (2005) 023003, arXiv:hep-th/0505215

[65] Joel K. Erickson, Daniel H. Wesley, Paul J. Steinhardt, Neil Turok, Kasner and Mixmaster behavior in universes with equation of state w $\geq$ 1, Phys.Rev. D69 (2004) 063514, arXiv:hep-th/0312009

[66] Subodh P. Patil, Exponential Inflation With $\rho = +p$, arXiv:hep-th/0509088

[67] I.Ya. Aref'eva, A.S. Koshelev, S.Yu. Vernov, Exactly Solvable SFT Inspired Phantom Model, arXiv:astro-ph/0412619

[68] Brett McInnes, De Sitter and Schwarzschild-De Sitter According to Schwarzschild and De Sitter, JHEP 09(2003)009, arXiv:hep-th/0308022

[69] Eva Silverstein, Dimensional Mutation and Spacelike Singularities, arXiv:hep-th/0510044

[70] Anastasios Psinas, Quantum Cosmology Aspects Of D3 Branes and Tachyon Dynamics, arXiv:hep-th/0511270

[71] Veronika E. Hubeny, Mukund Rangamani, Simon F. Ross, Causal structures and holography, JHEP 0507 (2005) 037, arXiv:hep-th/0504034

[72] John Horton Conway, Juan Pablo Rossetti, Describing the platycosms, arXiv:math.DG/0311476